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RF POWER LIMITATION ON MAIN INJECTOR BEAM CURRENT

INTRODUCTION

In this note we examine limits placed on the maximum beam current achievable in the Main Injector by the eighteen existing rf cavities with a single Y567B (4CW150,000) power amplifier tube installed in each cavity. The cavities are assumed to have R/Q ~ 120 and Q~ 6500 (at frequencies away from injection), giving R_{sh} ~7.8 x 10^5 Ohms. The cavities are expected to operate with effective accelerating voltage near 230 kV with voltage step-up ratio from anode to gap 12.25:1.

We assume that 'local' amplitude and phase feedback systems with bandwidth substantially larger than the synchrotron phase oscillation frequency are installed and operative so that the cavities may be detuned such that the power amplifier load appears 'real' without concern for Robinson stability or 'bucket area reduction factor'. (The presently proposed amplitude control system will probably be adequate. Additional phase feedback may be required as the cavity tuning system may not have sufficient bandwidth.) Also a relatively fast feed-forward system will be required to prevent rapid excursions of the rf phase and amplitude during gaps in the bunch train.

We expect that changes will be required in ancillary equipment such as solid state rf drive amplifiers, series tube modulators, anode or screen grid power supplies, or feedback systems when existing systems are found not to be adequate to allow the rf cavity with its existing power amplifier tube to reach maximum power capability.

The maximum acceleration rate is assumed to be 270 GeV/cs at 39 GeV and that bucket area ~1 eV/s is to be developed . Because the rf system is required to produce not only accelerating voltage and power but also the additional voltage necessary to create the requisite bucket area, it is important to examine the interaction of large intensity bunches with the reactive component of longitudinal beam pipe impedance[1,2]. Here we assume that Z/n for the ring is 3 Ohms inductive. Any real component of Z/n may affect the net accelerating voltage (and possibly cause fast transverse instability) but for lack of information we ignore this effect.

RF VOLTAGE AND POWER

For acceleration at 270 GeV/cs the required accelerating voltage $V_{ac}\sin\varphi_s$ is

$$V\sin\phi_s = \frac{2\pi R}{c}\frac{d(pc)}{dt} = \frac{270x10^9}{90.314x10^3} = 2.99x10^6 volts. \quad (1)$$

The rf power required is $P = \frac{e\beta c V\sin\phi_s}{2\pi R} = 4.32x10^{-8}$ watts per proton. (2)

The power amplifier tube is cathode driven with peak cathode rf voltage 540 volts; $V_c = -540\cos(\omega t)$. The control grid is grounded for rf but is dc biased (in this example) to -460 volts so that at t = 0 the control grid is driven 80 volts positive. When the cathode voltage reaches zero the grid voltage is -460 volts and the tube anode current is effectively cut off. The tube conducts over π radians of the rf cycle (class B operation). The screen grid voltage is set at +1500 volts so that the cathode to screen voltage varies during the conduction period from 2040 volts to 960 volts. The dc anode voltage is set at 20.5 kV and the anode rf swing is 18 kV so that the minimum instantaneous anode voltage reaches 2500 volts when the cathode voltage is at its negative maximum (i.e. the control grid is driven positive). The effective cathode to anode voltage is $V_{ca} = 20.5 - 17.46\cos(\omega t)$. The peak tube current is 95 A. By interpolating between constant current anode curves for 2000, 1500, 1250, and 1000 screen grid volts the instantaneous anode current is determined as a function of the conduction angle Θ (or ωt). The results are shown in Figure 1.

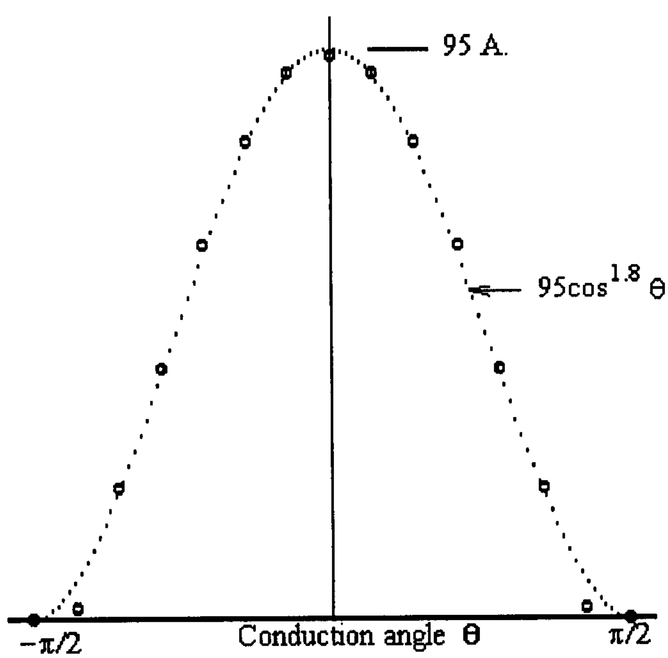

Figure 1. Circles are instantaneous cathode-anode current. Dotted curve is fit to the data by the function $95\cos^{1.8}\theta$.

The anode current is well represented over the conduction angle by the function $I(\theta) = 95\cos^{1.8}(\theta)$. The average anode current on one rf cycle is;

$$I_{av} = \frac{95}{\pi}\int_0^{\pi/2} \cos^{1.8}\theta d\theta = (0.26)(95) = 24.7A. \qquad \textbf{(3)}$$

The effective rf current delivered to the cavity by the current wave form is obtained through the Fourier transform of the function. The (normalized) first harmonic term is 0.83. The rf current delivered to the cavity is twice the normalized Fourier amplitude times the dc current. $I_{rf} = (2)(0.83)(24.7) = 41$ A. (Note that here the rf current is 1.66 times the average dc current. Data obtained many years ago by H.W. Miller suggested that the average rf current was near 1.1 times the dc current, giving a peak rf current factor of 1.56. Those data were obtained using a 50 Ohm water cooled load resistor so that the anode voltage was not allowed to swing more than a few kV, a basically different operation mode[3].)

$$P_a = \frac{V_d I_p}{\pi}\int_0^{\pi/2} \cos^{1.8}\theta(1 - 0.85\cos\theta)d\theta = 146\,kW. \qquad (4)$$

Under these conditions the tube anode dissipation isThe rf power delivered by the amplifier under these conditions is

$P_{rf} = \frac{1}{2}V_{rf}I_{rf} = (9 \times 10^3)(41) = 369$ kW. The anode efficiency is $\approx$ 72%, reasonable for class B.
With 18 kV anode rf voltage the cavity gap voltage is $V_g = (12.25)(18 \times 10^{3)}$ = 220.5 kV.
The cavity dissipation is $P_{cav} = V_g^2/2R_{sh} = (4.86 \times 10^{10})/(1.56 \times 10^6) = 31$ kW.
RF power available for acceleration is 369 - 31 = 338 kW per cavity or 6 MW for 18 cavities.

The available rf power divided by the power required per proton, Eq.(2), yields the maximum beam current capability. $N = 6 \times 10^6/(4.32 \times 10^{-8}) = 1.4 \times 10^{14}$ protons per cycle.

With these conditions the required cathode drive power will be $P_{cath} = \frac{(41)(540)}{2} = 11\,kW.$

<u>SCREEN GRID AND SERIES TUBE DISSIPATION</u>

Maximum screen grid current, 7 A., occurs when the anode voltage is minimum, and it decreases to a negligible value at 0.8 radians on either side of the maximum. The average screen current is 0.67 A. The screen grid voltage is near 2000 volts during the entire screen conduction period giving an average

If the anode supply voltage is set at 26.5 kV the voltage drop across the series modulator tube will be 6 kV. This should be adequate for control. The series tube dissipation will then be 150 kW.

LONGITUDINAL EMITTANCE; Z/N CONSIDERATIONS.

The total ring voltage is 4 MV with required accelerating voltage 3 MV. With $V_{ac} = V_t \sin\varphi_s$, $\sin\varphi_s \equiv \Gamma = 0.75$ and the 'moving bucket factor' $\alpha(\Gamma) = 0.125$. Ignoring for the moment beam chamber impedance effects, the bucket area becomes;

$$A_b = \alpha(\Gamma)\,\frac{8R}{hc}\left[\frac{2E_s V_{rf}}{\pi h \eta}\right]^{1/2} = (0.125)(7.9) = 1.03\ eV\text{-}s. \tag{5}$$

If the rf wave is considered to be traveling with the bunch then the voltage per turn can be expressed as a function of distance z along the rf period; $V_z = V_{rf}\sin(\frac{2\pi z}{\lambda})$ where λ is the rf wavelength. The longitudinal restoring force derives from the slope of the rf wave at the synchronous phase angle. The slope is $\frac{dV(z)}{dz} = V_{rf}\cos\phi_s = (4)(-0.661) = -2.64\,MV/m.$ ($\varphi_s = \pi - 0.848$)

1.4×10^{14} protons in 498 bunches is 2.8×10^{11} protons per bunch. Let B = protons per bunch. $\alpha(\Gamma) = 0.125$ sets a bucket length 0.7π radians or ≈ 2 m. The full bunch length may be $l = 1.5$ m. If a (worst case) parabolic line charge distribution is assumed, the moving bunch will induce in the vacuum chamber inductance per unit length a linear change in restoring potential which, above transition, must be subtracted from the restoring slope calculated above. Using expressions derived from Refs. 1,2, we find the maximum voltage induced by the beam at its edges (± 0.75 m.), assuming the ring Z/n =3 Ohms is;

$$V_b = \pm\frac{6eN\beta cC}{\pi l^3}\left|\frac{Z}{n}\right|\left[\frac{l}{2}\right] = \pm 5.7x\ 10^4\ volts, \qquad C = 3319\,m. \tag{6}$$

This amounts to a slope of 76 kV/m which is negligible compared to the rf generated slope calculated above. However, below transition, just following adiabatic capture, the beam induced force adds to the rf generated bucket parameters. Since the rf voltage at that point is limited to a substantially smaller value the beam induced potential may be more significant and it may be turned to advantage.

CONCLUSIONS

The rf equipment installed in the Main Injector enclosure appears to have the power capability to accelerate beam current in excess of 10^{14} protons per cycle at a peak acceleration rate of 260 GeV/cs. The solid state driver amplifiers will not be capable of generating the required drive power and the existing anode power supplies apparently do not have sufficient current delivery capability. Each of these deficiencies would appear to be correctable in the future without significant disruption of operation. The existing anode power supplies could easily be used to supply perhaps four stations each instead of six when additional supplies are made available. The solid state driver amplifiers could be upgraded one station at a time with only installation of new equipment in the galleries.

At intensity approaching 5×10^{14} protons per pulse the reactive component of ring impedance does not appear to represent a major problem as long as the ring Z/n is held below perhaps 5 Ohms. Addition of a substantial number of higher frequency rf cavities (~ 800 MHz perhaps) could cause some disruption and the problem should be studied carefully before proceeding.

REFERENCES

1. Theoretical Aspects of the Behavior of Beams in Accelerators and Storage Rings, CERN 77-13, (1977).
2. S. Hansen, H.G. Hereward, A. Hofmann, and K. Hübner, IEEE Trans. Nucl. Sci., NS-22, 1381, (1975)
3. H.W. Miller, Fermilab, Private communication, ca. 1975.

APPENDIX A - BEAM INTENSITY ACHIEVABLE WITH THREE PLANNED ANODE SUPPLIES

The maximum beam intensity achievable using only the three proposed anode power supplies and at a slightly decreased acceleration rate (240 GeV/cs) is calculated using the approach described above. Each of three anode supplies delivers 16 A. to each of six rf stations. (96 A. Total)

Accelerating voltage $V\sin\phi_s = F_\infty^{-1}\frac{d(pc)}{dt} = \frac{240 \times 10^9}{90.314 \times 10^3} = 2.66\,MV\,per\,turn.$

Maximum ring voltage 3.6 MV per turn.

$\Gamma = \sin\varphi_s = 0.739$, $\alpha(\Gamma) = 0.140$, A(0) = 7.53 eV-s, A(Γ) = (0.140)(7.53) = 1.05 eV-s.

Gap voltage per cavity V_g = 200 kV. Anode voltage V_{rf} = 16.33 kV peak.

The dc anode voltage is set at 19 kV so the anode voltage swings down to 2.67 kV.

Screen voltage set at 1500 vdc. The cathode is driven at $V_c = 400\cos\omega t$ and control grid set at -430 vdc.

Peak cathode current is 64 A. The current wave is well represented by $64\cos^2\Theta$. Conduction angle $\pi/2$.

Fundamental Fourier component is (1.69) I_{av} = 27 A. $I_{av} = 0.25\, I_p$ = 16 A.

RF power output $P_{rf} = \frac{V_{rf} I_{rf}}{2} = \frac{(16.33 x 10^3)(27)}{2} = 221\ kW.$

RF cavity dissipation, $P_{cav} = \frac{V_g^2}{2R_{sh}} = \frac{(2 \times 10^5)^2}{1.56 \times 10^6} = 25.6\,kW.$

Anode dissipation, $P_a = \frac{I_p V_a}{\pi}\int_0^{\pi/2} \cos^2\theta\,(1 - 0.84\cos\theta)\,d\theta = I_p V_a (0.072) = (64)(19 x 10^3)(0.072) = 87.6\ kW.$

Peak screen current I_{psg} = 4 A. Avg. Screen current I_{asc} = 0.44 A. Screen dissipation, 880 Watt..

With anode supply voltage 26.5 kV the series tube dissipation is 120 kW.

RF power available for acceleration, (18 cavities); P_{rf} = (18)(221 - 25.6) = 3.52 MW.

Required beam acceleration power, $P_{ac} = \frac{e\beta c V_{ac}}{C} = 3.85 x 10^{-8}$ *Watts perproton.*

Achievable beam intensity, N = 9.1 x 10^{13} protons per cycle.*

*The presently proposed 4 kW cathode driver amplifiers are not sufficient to reach this intensity.